# Retail Reverse Supply Chain Management and Inventory Optimization

Computational Science and Engineering
&
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# Forward Supply Chain vs Reverse Supply Chain

**Forward Supply Chains**

- Material flow from FEW central locations to MANY demand points which is divergent.

(such as factories, plants, suppliers, vendors, etc. to warehouses, stores, customers, etc.)

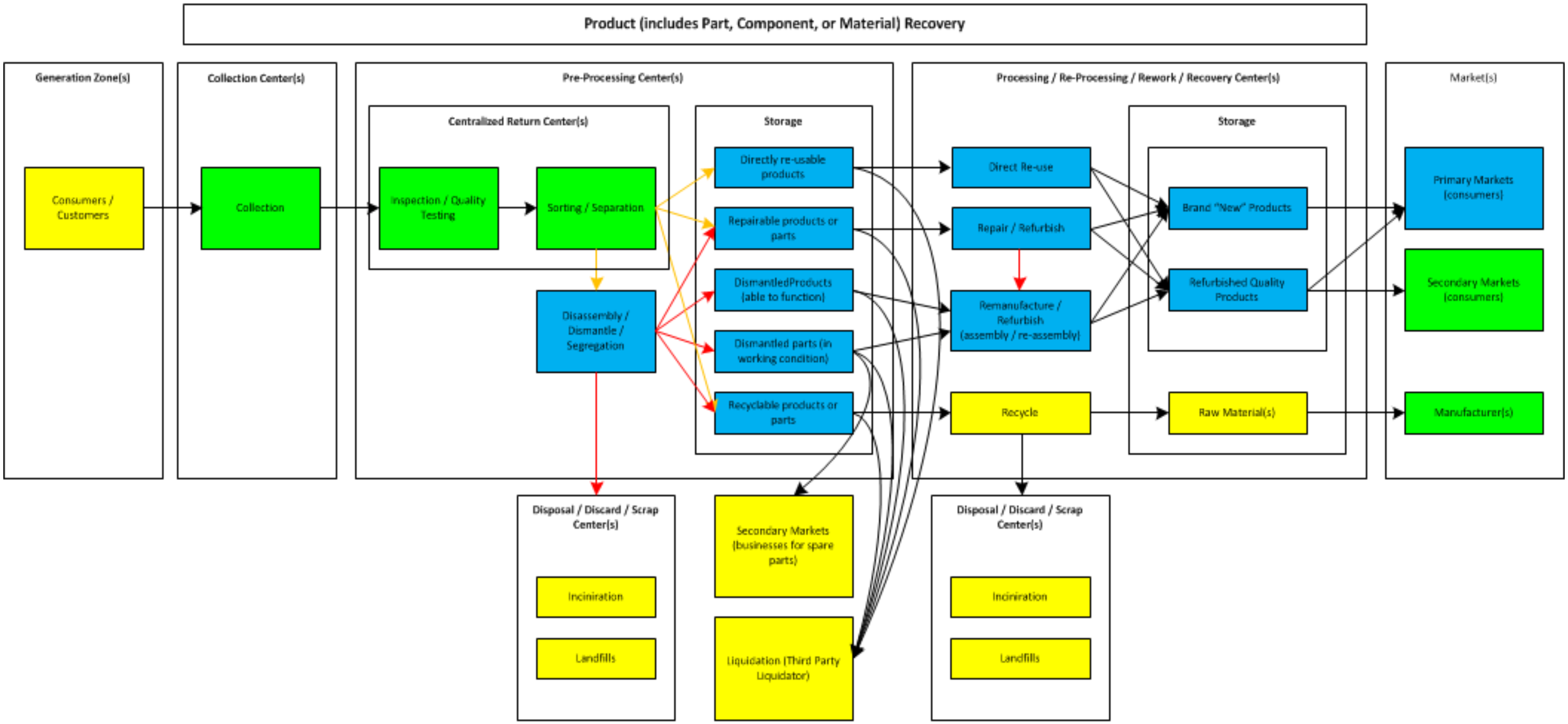
**Reverse Supply Chains**

- Material flow from MANY collection points to FEW demand centers which is convergent.

(such as consumers, customers, refuse bins, collection points, stores, etc. to suppliers, factories, plants, vendors)

**Reverse Logistics**: Deals with material/product flow from point of consumption to the point of origin to recapture value or dispose properly.

# Reverse Supply Chain Management



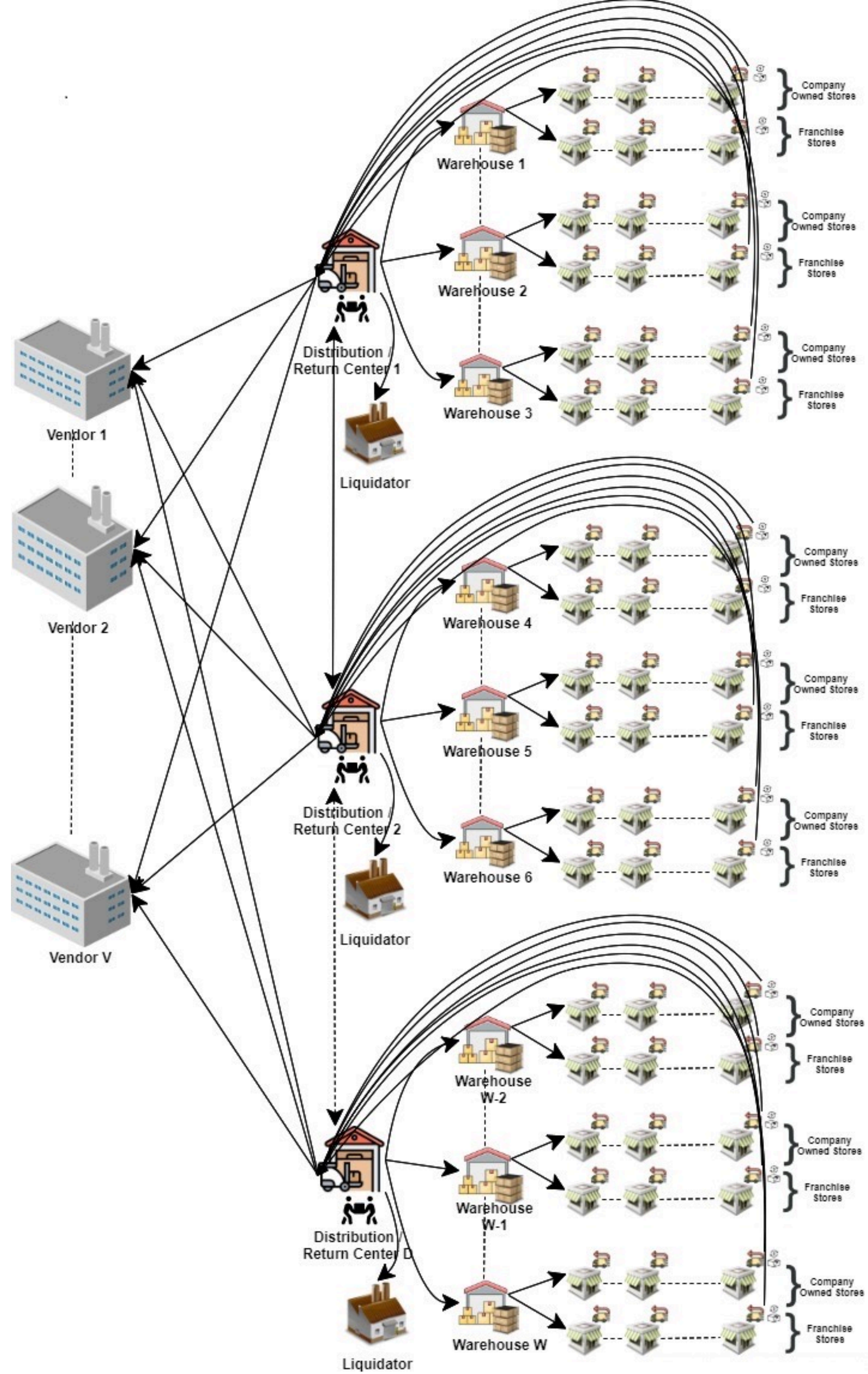

This color coding represents that facilities are owned by the manufacturer.

This color coding represents that facilities are NOT owned by the manufacturer.

This color coding represents that facilities can be owned by the manufacturer or some other manufacturer, business or third party logistics provider.

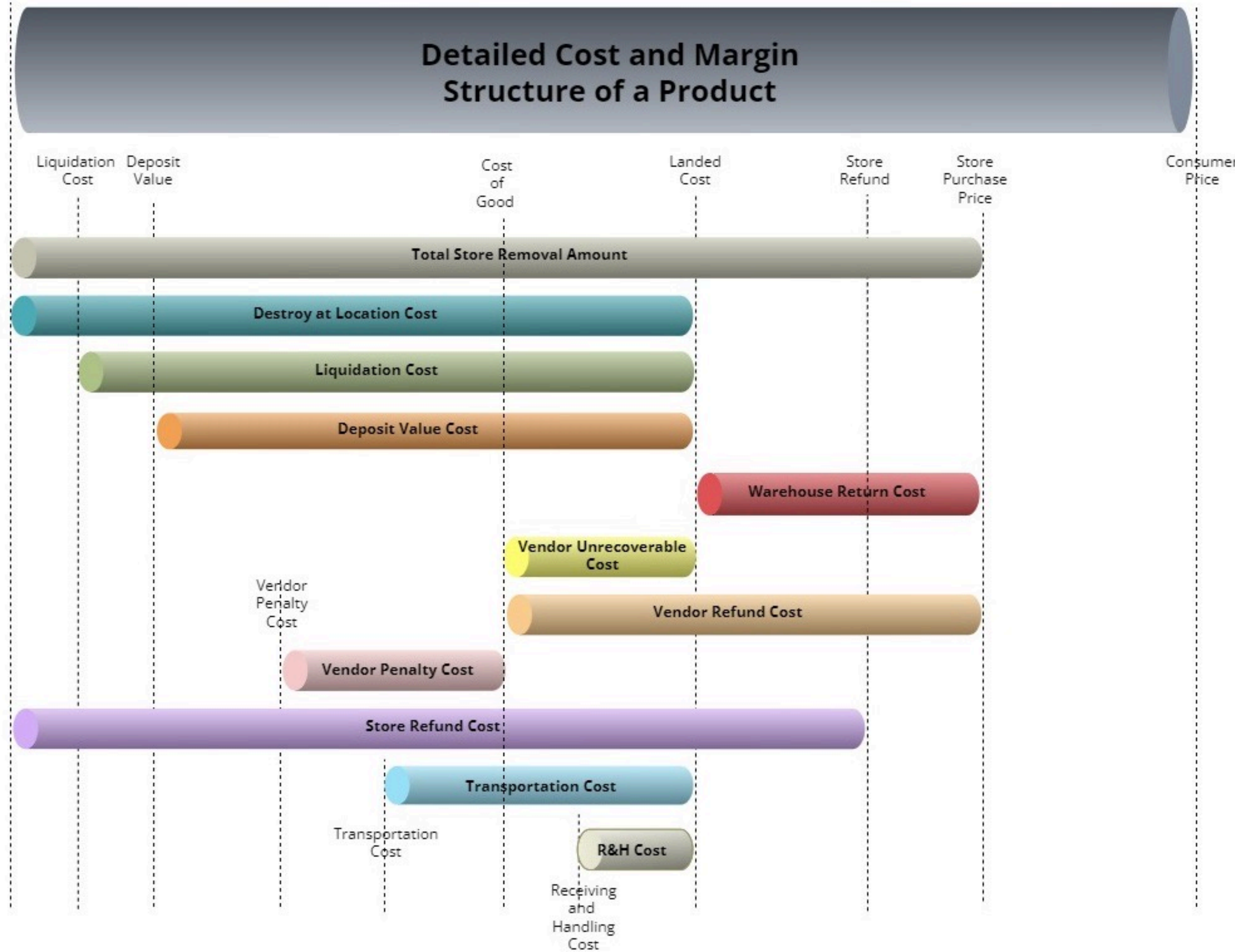
A General Framework for Possible Reverse Logistics Activities and Reverse Supply Chain Facilities

Retail Reverse Supply Chain (RRSC) Network



# Problem Definition

- Company is the ultimate decision maker.
- Stores hold excessive inventory.
- Pull a certain amount of inventory from all stores.
- Only some of products (ineffective inventory) will be recalled.
- Excess inventory will be routed.
- **objective** is to minimize all supply chain and inventory related costs
- **subject to** capacity, vendor funds, warehouse/store demand and rigid RRSC network structure, etc.




+ Plant Activation Cost
(Not a part of the above cost structure of a product)

# Mathematical Optimization Model for RRSC Decisions

- **Decision Variables**
  - Which store-products will be chosen to be disposed. (product selection)
  - If a product is chosen from a store, where should it go. (product allocation)
  - Which Distribution/Return Centers should be activated. (facility selection)
- **Multi-Objective**
  - Minimize:
    - Destroying and liquidating products and costs related to destroying or liquidating.
    - Profit margin losses and penalties because of vendor returns.
  - Maximize:
    - Warehouse Returns with most profitable products.
    - Vendor Returns with least profitable products.
- **Constraints**
  - Pulling a certain amount of inventory from stores.
  - Actions that can be taken for a store-product.
  - Logical rules that satisfy the product movement structure within the complex retail reverse supply chain network.
  - Capacity, demand for warehouse and funds available for vendors returns.

# Inventory Optimization Model for Retail RSC Decisions

$$\text{minimize z=} \; DestroyAtSiteCost + LiquidationCost + WarehouseReturnCost + DepositValueCost + VendorRefundCost + VendorRefundUnrecoverableCost + VendorPenaltyCost + TotalStoreRefund + TotalStoreRemoval + TotalTransportationCost + Receving\&HandlingCost + PlantActivationCost$$

$$TotalTransportationCost = TransportationCostFromStoresToReturnCenters + TransportationCostAmongReturnCenters + TransportationCostFromReturnCentersToVendors + TransportationCostFromReturnCentersToWarehouses + TransportationCostFromReturnCentersToLiquidation$$



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$$\text{minimize z=} \sum_{d=1}^{D} ac_d \, DAC_d \quad +$$

→ Plant Activation cost,

- Cost of activating distribution centers for reverse supply chain activities.

$$\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{d=1}^{D}(1 - rt_{s,p,d}) \, Qty_{s,p} \, E_{s,d} \, WE_p/VOL_p \, RHF_d +$$

$$\sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{d' \in D, \text{where } d' \neq d} tr_{p,d,d'} \, E_{d,d'} \, WE_p/VOL_p \, RHF_{d'}$$

Receiving & Handling cost,

- Cost of receiving and handling products at the distribution centers that are coming from
  - Stores
  - Transfers from other distribution centers.

s.t. …

# Illustrative Example

- 4 Distribution Centers,
  - 2 Warehouse Each
- 8 Warehouses,
- 500 Stores,
- 2 Store Types,
  - Company Owned
  - Franchise
- 1,000 unique Products,
- 10,000 Store-Products,
  - an average of 20 products per store.
- 20 Vendors.

- 60,004 binary
- 68,080 integer decision variables

```
CPXPARAM_TimeLimit                        36000
CPXPARAM_MIP_Tolerances_MIPGap            0.01
CPXPARAM_MIP_Strategy_File                3
CPXPARAM_Emphasis_MIP                     1
```

```
Elapsed time = 2522.63 sec. (982776.69 ticks, tree = 6218.00 MB, solutions = 307)
Nodefile size = 4074.31 MB (1859.70 MB after compression)
* 36529+ 6997                         527821.8647   522010.4502            1.10%
* 36531+ 6997                         527761.8571   522010.4502            1.09%
* 36535+ 6997                         527757.5215   522010.4502            1.09%
  36649  7099   526004.3566    97     527757.5215   522010.4502  1594084   1.09%
  36880  6539   526965.8350   104     527757.5215   522010.4502  1601308   1.09%
  37038  6615   527432.1418    95     527757.5215   522010.4502  1601643   1.09%
  37132  6691   527617.7967   103     527757.5215   522010.4502  1602252   1.09%
  37168  7079   522486.1893  1698     527757.5215   522064.0638  1610823   1.08%

Cover cuts applied:  10
Implied bound cuts applied:  17
Flow cuts applied:  37
Mixed integer rounding cuts applied:  72
Zero-half cuts applied:  1
Gomory fractional cuts applied:  4

Root node processing (before b&c):
  Real time             =   32.92 sec. (20091.62 ticks)
Parallel b&c, 4 threads:
  Real time             = 2544.13 sec. (982727.25 ticks)
  Sync time (average)   =  624.31 sec.
  Wait time (average)   =    0.09 sec.
                          ------------
Total (root+branch&cut) = 2577.04 sec. (1002818.86 ticks)
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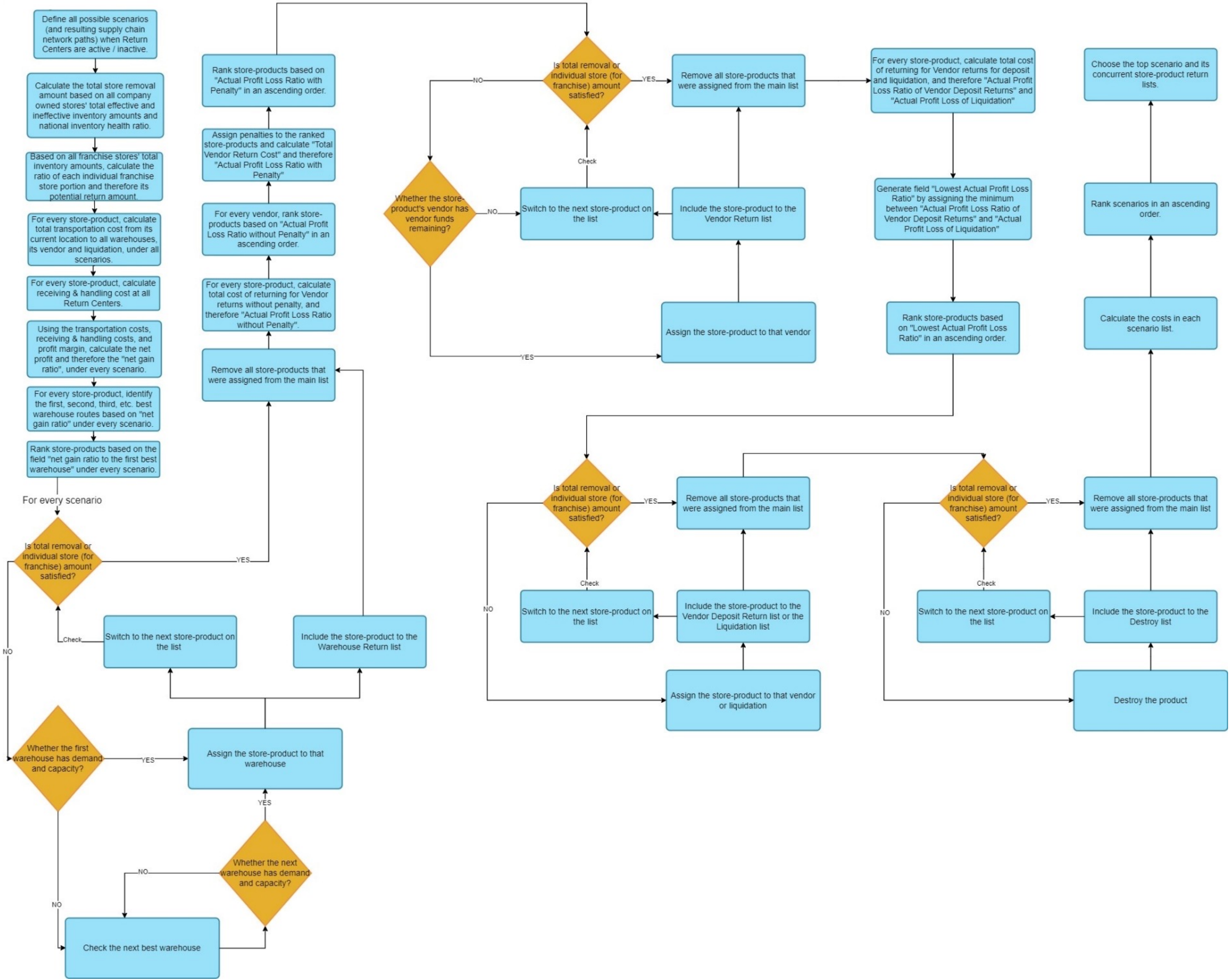
# Heuristic Algorithm



# Test Results, CPLEX vs. Heuristic

- Small-to-large size test problems are solved with both CPLEX and Heuristic.
- Heuristic found close-to-optimal solutions in reasonable times.

| Test Case | Problem Size (# of Store - Products) | Solution Methodology Used | Optimality Range Found within the Given Time | Solution Time Given or Found (seconds) | Solution Time Given or Found (minutes) | Solution Time Given or Found (hours) | TOTAL COST of REVERSE SUPPLY CHAIN ACTIVITY |
|---|---|---|---|---|---|---|---|
| Test 01 | | CPLEX | 2.31% | 1,201 | 20 | 0.33 | $ 779,029 |
| | 10,000 | Heuristic | 2.50% | 1,163 | 19.23 | 0.32 | $ 780,587 |
| Test 02 | | CPLEX | 2.76% | 1,200 | 20 | 0.33 | $ 706,012 |
| | 10,000 | Heuristic | 3.24% | 1,106 | 18.26 | 0.31 | $ 709,513 |
| Test 03 | | CPLEX | 3.84% | 3,603 | 60 | 1.00 | $ 583,865 |
| | 10,000 | Heuristic | 3.73% | 1,084 | 18.04 | 0.31 | $ 583,168 |
| Test 04 | | CPLEX | 1.26% | 3,603 | 60 | 1.00 | $ 870,638 |
| | 10,000 | Heuristic | 3.71% | 1,237 | 20.37 | 0.34 | $ 892,832 |
| Test 05 | | CPLEX | 1.12% | 3,603 | 60 | 1.00 | $ 1,141,319 |
| | 10,000 | Heuristic | 2.42% | 1,313 | 21.53 | 0.36 | $ 1,156,477 |
| Test 06 | | CPLEX | 3.78% | 7,202 | 120 | 2.00 | $ 853,607 |
| | 25,000 | Heuristic | 4.58% | 3,375 | 56.15 | 0.94 | $ 860,758 |
| Test 07 | | CPLEX | 1.84% | 10,804 | 180 | 3.00 | $ 917,947 |
| | 25,000 | Heuristic | 4.68% | 3,429 | 57.09 | 0.95 | $ 945,268 |
| Test 08 | | CPLEX | 2.16% | 7,208 | 120 | 2.00 | $ 1,042,709 |
| | 25,000 | Heuristic | 4.42% | 3,485 | 58.05 | 0.96 | $ 1,067,373 |
| Test 09 | | CPLEX | 4.05% | 7,216 | 120 | 2.00 | $ 1,029,828 |
| | 25,000 | Heuristic | 5.53% | 3,481 | 58.01 | 0.96 | $ 1,045,929 |
| Test 10 | | CPLEX | 5.98% | 7,201 | 120 | 2.00 | $ 1,293,328 |
| | 25,000 | Heuristic | 5.65% | 3,626 | 60.26 | 1.00 | $ 1,288,861 |
| Test 11 | | CPLEX | 1.19% | 28,817 | 480.3 | 8.00 | $ 2,617,198 |
| | 50,000 | Heuristic | 6.02% | 8,280 | 138 | 2.18 | $ 2,751,560 |
| Test 12 | | CPLEX | 0.99% | 24,940 | 415.6 | 6.93 | $ 2,324,932 |
| | 50,000 | Heuristic | 4.62% | 9,000 | 150 | 2.30 | $ 2,413,525 |
| Test 13 | | CPLEX | 2.02% | 14,412 | 240.2 | 4.00 | $ 2,337,156 |
| | 50,000 | Heuristic | 3.74% | 8,100 | 135 | 2.15 | $ 2,378,925 |
| Test 14 | | CPLEX | 1.80% | 14,239 | 237 | 3.95 | $ 2,109,302 |
| | 50,000 | Heuristic | 6.25% | 7,740 | 129 | 2.09 | $ 2,201,606 |
| Test 15 | | CPLEX | 2.86% | 11,244 | 187.4 | 3.12 | $ 1,906,949 |
| | 50,000 | Heuristic | 3.18% | 7,560 | 126 | 2.06 | $ 1,913,348 |
| Test 16 | | CPLEX | 1.13% | 28,806 | 480 | 8.00 | $ 4,149,720 |
| | 100,000 | Heuristic | 0.88% | 16,260 | 271 | 4.31 | $ 4,139,719 |
| Test 17 | | CPLEX | 1.09% | 28,802 | 480 | 8.00 | $ 5,745,484 |
| | 100,000 | Heuristic | 0.38% | 18,900 | 315 | 5.15 | $ 5,704,475 |
| Test 18 | | CPLEX | 0.88% | 24,059 | 400.1 | 6.68 | $ 6,108,926 |
| | 100,000 | Heuristic | 1.02% | 19,920 | 332 | 5.32 | $ 6,117,818 |
| Test 19 | | CPLEX | 1.41% | 25,206 | 420 | 7.00 | $ 4,502,148 |
| | 100,000 | Heuristic | 0.26% | 16,620 | 277 | 4.37 | $ 4,450,337 |
| Test 20 | | CPLEX | 1.62% | 6,222 | 103 | 1.72 | $ 6,587,367 |
| | 100,000 | Heuristic | 0.21% | 20,820 | 347 | 5.47 | $ 6,468,131 |

# Test Results, CPLEX vs. Heuristic

- Small-to-large size test problems are solved with both CPLEX and Heuristic.
- Heuristic found
  - close-to-optimal (0.2-5% optimality range) solutions
  - in reasonable times (20 minutes to 5 hours) .
- For small size problems,
  - Heuristic and CPLEX performed similarly.
- For large size problems,
  - Heuristic found better solutions in shorter time.

# Insights

- Model solutions suggests that
  - For realistic size RRSC problems, state-of-the-art solvers take significant time and memory to solve these kinds of problems.
  - A retail company can use our multi-stage heuristic to make inventory decisions for realistic size problems which would provide close-to-optimal solutions.

# Appendix

# Inventory Optimization Model for Retail RSC Decisions

$$\text{minimize z=} \begin{array}{l} DestroyAtSiteCost + LiquidationCost + WarehouseReturnCost + \\ DepositValueCost + VendorRefundCost + VendorRefundUnrecoverableCost + \\ VendorPenaltyCost + TotalStoreRefund + TotalStoreRemoval + \\ \boxed{TotalTransportationCost} + Receving\&HandlingCost + PlantActivationCost \end{array}$$

$$TotalTransportationCost = \begin{array}{l} TransportationCostFromStoresToReturnCenters + \\ TransportationCostAmongReturnCenters + \\ TransportationCostFromReturnCentersToVendors + \\ TransportationCostFromReturnCentersToWarehouses + \\ TransportationCostFromReturnCentersToLiquidation \end{array}$$



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$$\text{minimize z=} \sum_{d=1}^{D} ac_d \, DAC_d \quad +$$

→ Plant Activation cost,

- Cost of activating distribution centers for reverse supply chain activities.

$$\left.\begin{array}{l} \sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{d=1}^{D}(1 - rt_{s,p,d}) \, Qty_{s,p} \, E_{s,d} \, WE_p/VOL_p \, RHF_d + \\ \sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{d' \in D, \text{where } d' \neq d} tr_{p,d,d'} \, E_{d,d'} \, WE_p/VOL_p \, RHF_{d'} \end{array}\right\}$$

Receiving & Handling cost,

- Cost of receiving and handling products at the distribution centers that are coming from
  - Stores
  - Transfers from other distribution centers.

# Inventory Optimization Model for Retail RSC Decisions

$$\sum_{s=1}^{S}\sum_{p=1}^{P^-} ds_{s,p}Qty_{s,p}LC_p \quad +$$

→ Cost of destroying products at Site
- If products need to be destroyed, destroy invaluable products

$$\sum_{p=1}^{P^-}\sum_{d=1}^{D} lq_{p,d}\left(LC_p - COG_pLQR_p\right) \quad +$$

→ Liquidation cost
- Minimum lost from liquidating products.

$$\sum_{p=1}^{P^-}\sum_{d=1}^{D}\sum_{w=1}^{W} wh_{p,d,w}\left(LC_p - SP_p\right) \quad +$$

→ Warehouse returns,
- maximum warehouse returns with the most profitable products.

$$\sum_{v=1}^{V}\begin{cases} VPF1_v\, vf_v & \text{if } 0 \le vf_v \le RT1_v \\ VPF1_v\, RT1_v + VPF2_v\left(vf_v - RT1_v\right) & \text{if } RT1_v < vf_v \le RT2_v \\ VPF1_v\, RT1_v + VPF2_v\left(RT2_v - RT1_v\right) & \text{if } RT2_v < vf_v \le VF_v \\ +VPF3_v\left(vf_v - RT2_v\right) & \end{cases} \quad +$$

→ Vendor Penalty Cost,
- minimum penalty cost from returning a product to its vendor.

$$\sum_{p=1}^{P^-}\sum_{st\in ST}\sum_{d=1}^{D} dp_{p,st,d}(LC_p - DV_p) \quad +$$

→ Vendor deposit-refund cost,
- minimum margin lost from deposit return.

$$\sum_{s=1}^{S}\sum_{p=1}^{P^-}\left(ds_{s,p}SP_pQty_{s,p}R_p + (1 - rt_{s,p})SP_pQty_{s,p}R_p\right) \quad +$$

→ Store Refunds

# Inventory Optimization Model for Retail RSC Decisions

$$\sum_{p=1}^{P^-} \sum_{st \in ST} \sum_{d=1}^{D} vr_{p,st,d} \, (LC_p - COG_p) \; +$$

→ Vendor refund unrecoverable cost,

- minimum lost from exchange and shipping costs.
- cost of doing business.

$$\sum_{p=1}^{P^-} \sum_{st \in ST} \sum_{d=1}^{D} vr_{p,st,d} \, (SP_p - COG_p) \; +$$

→ Vendor refund cost,

- minimum lost from profit margin.

$$\left( \sum_{p=1}^{P-} \sum_{s=1}^{S} \sum_{d=1}^{D} (1 - rt_{s,p,d}) \, Qty_{s,p} \, E_{s,d} \, VOL_p \, TCD_{s,d} \right) \; +$$

→ Transportation cost,

- From Stores to Distribution Centers.

$$\left( \sum_{p=1}^{P-} \sum_{d=1}^{D} \sum_{d'=1}^{D} tr_{p,d,d'} E_{d,d'} VOL_p \, TCT_{d,d'} \right) \; +$$

→ Transportation cost,

- Distribution Centers to Distribution Centers transfers.

$$\left( \sum_{p=1}^{P-} \sum_{st \in ST} \sum_{d=1}^{D} vr_{p,st,d} VOL_p \, TCV_{d,v} \right) +$$

$$\left( \sum_{p=1}^{P-} \sum_{st \in ST} \sum_{d=1}^{D} dp_{p,st,d} VOL_p \, TCV_{d,v} \right) \; +$$

} Transportation cost,

- From Distribution Centers to Vendors.

$$\left( \sum_{p=1}^{P-} \sum_{w=1}^{W} \sum_{d=1}^{D} wh_{p,d,w} \, E_{d,w} \, VOL_p \, TCW_{d,w} \right) \; +$$

→ Transportation cost,

- From Distribution Centers to Warehouses

$$\left( \sum_{p=1}^{P-} \sum_{d=1}^{D} lq_{p,d} \, VOL_p \, TCL_d \right) \; +$$

→ Transportation cost,

- From Distribution Centers to Liquidation.

# Inventory Optimization Model for Retail RSC Decisions

$$\text{s.t.} \quad ds_{s,p} \leq rt_{s,p} \qquad \forall\, s \in S, p \in P^-$$

Possible actions to be taken for a store-product,

- Do nothing, leave the product at store
- Destroy the product at store
- Return the product to distribution center

$$\sum_{s=1}^{S}(1 - rt_{s,p})Qty_{s,p} = \sum_{s \in ST}\sum_{d=1}^{D} vr_{p,st,d} + \sum_{s \in ST}\sum_{d=1}^{D} dp_{p,st,d} + \sum_{d=1}^{D} lq_{p,d} + \sum_{d=1}^{D}\sum_{w=1}^{W} wh_{p,d,w} \qquad \forall\, p \in P^-$$

Total number of items for a products that are being returned is equal to the items that are going to related destinations.

$$\sum_{d=1}^{D} wh_{p,d,w} \leq WDC_{p,w} \qquad \forall\, w \in W, p \in P^-$$

Warehouse return for a product cannot exceed product demand from other stores.

$$\sum_{p=1}^{P^-}\sum_{d=1}^{D} wh_{p,d,w} COG_p \leq WRA_w \qquad \forall\, w \in W$$

Warehouse return value for all products cannot exceed warehouse return allowance budget.

$$\sum_{p=1}^{P^-}\sum_{d=1}^{D} vr_{p^{(v)},st',d}\, COG_p = \sum_{st \in ST} vf_{v,st,st'} \qquad \forall\, v \in V, st \in ST$$

Define vendor funds used from a specific store types' vendor funds.

$$\sum_{st' \in ST} vf_{v,st,st'} \leq VF_{v,st} \qquad \forall\, v \in V, st \in ST$$

Vendor return used for all products cannot exceed total vendor funds for each vendor.

# Inventory Optimization Model for Retail RSC Decisions

$$1 - rt_{s,p} = \sum_{d=1}^{D} (1 - rt_{s,p,d}) \qquad \forall\, s \in S, p \in P^{-}$$

→ A Store-Product can only be returned to one distribution center.

$$rt_{s,p} \leq rt_{s,p,d} \qquad \forall\, s \in S, p \in P^{-}, d \in D$$

→ Defining logic of return a store-product to a distribution center.

$$1 - rt_{s,p,d} \leq ac_d \qquad \forall\, s \in S, p \in P^{-}, d \in D$$

→ If a distribution center is NOT activated, then the stores can NOT return products to this distribution center.

$$E_{s,d}\left(rt_{s,p} - rt_{s,p,d} - ac_d\right) \geq -1 \qquad \forall\, s \in S, p \in P^{-}, d \in D$$

→ Defining logic of activating a distribution center and related return rules.

$$\sum_{s=1}^{S} (1 - rt_{s,p,d}) Qty_{s,p} + \sum_{d \in D, \text{where } d \neq d'} tr_{p,d,d'}$$
$$= \sum_{s \in ST} vr_{p,st,d} + \sum_{s \in ST} dp_{p,st,d} + lq_{p,d} + \sum_{w=1}^{W} wh_{p,d,w}\, E_{d,w} +$$
$$\sum_{w=1}^{W} wh_{p,d,w}\,(1 - E_{d,w}) + \sum_{d' \in D, \text{where } d' \neq d} tr_{p,d,d'} \qquad \forall\, p \in P^{-}, d \in D$$

→ Total number of items for a products that are coming in to DC is equal to the items that are going out from DC.

$$\sum_{d \in D, \text{where } d \neq d'} tr_{p,d,d'} \leq ac_{d'}\; M \qquad \forall\, p \in P^{-}, d \in D$$

→ If a DC is NOT activated, it can NOT receive transfers from other related DCs

$$\sum_{d' \in D, \text{where } d' \neq d} tr_{p,d,d'} \leq ac_d\; M \qquad \forall\, p \in P^{-}, d \in D$$

→ If a DC is NOT activated, it can NOT send transfers from other related DCs.

$$\sum_{d \in D, \text{where } d \neq d'} tr_{p,d,d'} \leq \sum_{w=1}^{W} wh_{p,d,w} + \sum_{d'=1}^{D} tr_{p,d,d'} \qquad \forall\, p \in P^{-}, d \in D$$

→ Received transfers cannot be sent to vendors or liquidation

# Inventory Optimization Model for Retail RSC Decisions

$$tr_{p,d,d'} \geq 1 \rightarrow tr_{p,d',d} \leq 0 \qquad \forall\, p \in P^-, d \in D, d' \in D$$

→ If there is transfer among distribution centers for a product, then reverse path should not exist.

$$\sum_{st' \in ST, st' = st} vf_{v,st,st'} \leq RB1_{v,st} \rightarrow \sum_{st' \in ST, st' \neq st} vf_{v,st,st'} \leq 0 \qquad \forall\, v \in V, st \in ST$$

→ Compatible vendor funds can only be transferred/used if the available vendor funds for a store type does not cover to return those items to its vendor.

$$\sum_{d' \in D, \text{where } d' = d} tr_{p,d,d'} = 0 \qquad \forall\, p \in P^-, d \in D$$

→ A distribution center should not transfer items of a product to itself

$$tr_{p,d,d'} \leq E_{d,d'} \ M \qquad \forall\, p \in P^-, d \in D, d' \in D$$

→ A distribution center can only transfer items of a product to another DC if a path exits.

$$(1 - rt_{s,p,d}) \leq E_{s,d} \ M \qquad \forall\, s \in S, p \in P^-, d \in D$$

→ A store can only send items of a product to DC if a path exits.

$$wh_{p,d,w} \leq E_{d,w} \ M \qquad \forall\, p \in P^-, d \in D, w \in W$$

→ A distribution center can only send items of a product to warehouse if a path exits.